Targeted Learning for Data Adaptive Causal Inference in Observational and Randomized Studies

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Outline of estimation part of course

- Part 1
  - Targeted Learning Overview
  - Estimation Roadmap
  - Super Learning
- Part 2
  - Targeted Minimum Loss-Based Estimation (TMLE)
- Part 3
  - TMLE for longitudinal data analysis
  - Concluding Remarks

Estimation is a Science, Not an Art

- Data: Realizations of random variables with probability distribution P_0
- Statistical Model ($\mathcal{M}$): Actual knowledge about the shape of P_0
- Statistical Target Parameter: A feature/function of P_0
- Estimator
  - Algorithm for mapping from data to a (d-dimensional) real number
  - Benchmarked by a dissimilarity measure (e.g., MSE) w.r.t. target parameter

Causal Inference

- Under non-testable assumptions P_0 can be described in terms of an underlying parameter varying over an underlying parameter space
  - e.g., intervention-specific counterfactuals
  - Parameter space described by a full data model such as a nonparametric structural equation model (NPSEM)
- Non-testable assumptions enrich the interpretation of the statistical target parameter
  - Model = Statistical Model + Untestable Assumptions
  - Allows identification of full data parameters and causal quantities
  - Defintion of statistical target parameter is clear. Causal interpretation when assumptions are met
- **Statistical estimation is concerned only with statistical model and statistical target parameter**

Example: TMLE for the ATE parameter

Marginal additive effect of a binary point treatment (ATE) parameter

- Data: n i.i.d. observations $O = (Y, A, W) \sim P_0$

  outcome Y, binary treatment indicator A, covariate vector W

- NPSEM

$$\begin{aligned} W &= f_W(U_W) \\ A &= f_A(W, U_A) \\ Y &= f_Y(W, A, U_Y) \end{aligned}$$

  where U_W, U_A, U_Y are uncorrelated exogenous random errors

- Target Parameter: $\psi_0^{ATE} = E_0(Y_1 - Y_0)$
  - Y_a is counterfactual outcome generated under NPSEM with A set to a

A Substitution Estimator

- Target parameter ψ_0 is a feature of P_0
  - ψ_0 can be expressed as a mapping, $\Psi(P_0)$
  - ψ_0 is sometimes a feature of only a portion of P_0 denoted by Q_0
  - Thus, $\Psi(Q_0) = \Psi(P_0)$
- A substitution estimator applies the target parameter mapping directly to an estimate of relevent component
  - Q_n is an estimator of Q_0 that conforms to statistical model $\mathcal{M}$
  - Substitution estimator can be represented as mapping $\psi_n = \Psi(Q_n)$.
- Use of a substitution estimator enhances robustness by respecting bounds on both $\mathcal{M}$ and ψ_0

TMLE for the ATE parameter

- Likelihood factorizes

$$\mathcal{L}(O) = \underbrace{P(Y \mid A, W)}_{Q_Y} \underbrace{P(A \mid W)}_{g} \underbrace{P(W)}_{Q_W}$$

- Define

$$Q_0 = (Q_{0_Y}, Q_{0_W})$$
$$g_0 = P_0(A \mid W)$$

- When causal assumptions are met ψ_0^{ATE} equals

$$\begin{aligned}\psi_0 &= E_{0_W}[E_0(Y \mid A = 1, W) - E_0(Y \mid A = 0, W)] \\ &= E_{0_W}\left[\bar{Q}_{0_Y}(1, W) - \bar{Q}_{0_Y}(0, W)\right]\end{aligned}$$

- Otherwise, ψ_0 remains a useful measure of variable importance

Motivation for TMLE

- Super Learner (SL) for estimating Q_0
- SL-based substitution estimator evaluates $\psi_n^{SL} = \Psi(Q_n)$
- TMLE fluctuates initial Q_n to obtain targeted Q_n^*
  - Targeting makes use of information in P_0 beyond Q_0 to improve estimation of ψ_0
  - Provides an opportunity to
    - reduce asymptotic bias if initial Q_n not consistent
    - reduce finite sample bias
    - reduce variance

Pathwise Differentiable Parameter

- Pathwise derivative for a path $\{P(\epsilon) : \epsilon\} \subset \mathcal{M}$ through P at $\epsilon = 0$ is defined by $\frac{d}{d\epsilon}\Psi(P(\epsilon))|_{\epsilon=0}$
- If for all paths through P this derivative can be represented as

$$P(D^*(P)S) \equiv \int D^*(P)(o)S(o)dP(o),$$

where S is the score of the path at $\epsilon = 0$,

and $D^*(P)$ is an element of tangent space at P,

then

  - the target parameter mapping is pathwise differentiable at P
  - its canonical gradient (i.e., efficient influence curve), is $D^*(P)$

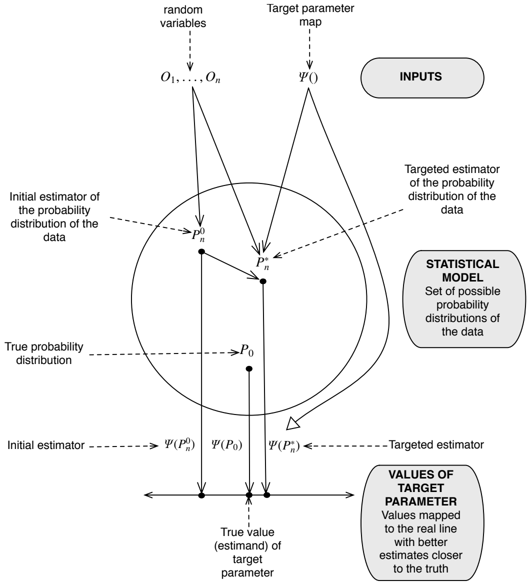
Efficient Influence Curve Equation

- An estimator is asymptotically efficient if and only if it is asymptotically linear with influence curve the efficient influence curve $D^*(P_0)$:

$$\psi_n - \psi_0 = \frac{1}{n} \sum D^*(P_0)(O_i) + o_P\left(\frac{1}{\sqrt{n}}\right)$$

- An efficient regular asymptotically linear estimator will need to solve the efficient influence curve equation $\sum_i D^*(P)(O_i) = 0$ (up to second order term)
- TMLE is a double robust semi-parametric efficient RAL substitution estimator that can be applied to estimate any pathwise differential parameter of P_0.

TMLE



random variables
Target parameter map
$O_1, \ldots, O_n$
$\Psi()$
INPUTS
Initial estimator of the probability distribution of the data
Targeted estimator of the probability distribution of the data
P_n^0
P_n^*
STATISTICAL MODEL
Set of possible probability distributions of the data
True probability distribution
P_0
Initial estimator
$\Psi(P_n^0)$
$\Psi(P_0)$
$\Psi(P_n^*)$
Targeted estimator
VALUES OF TARGET PARAMETER
Values mapped to the real line with better estimates closer to the truth
True value (estimand) of target parameter

Targeted Minimum Loss-Based Estimation

- TMLE Procedure
  1. Identify the "hardest" parametric submodel to fluctuate initial $\hat{P}$
     Small fluctuation $\rightarrow$ maximum change in target
  2. Identify optimal magnitude of fluctuation by MLE
  3. Apply optimal fluctuation to $\hat{P}$ to obtain 1st-step TMLE
  4. Repeat until incremental fluctuation is zero
     - 1-step convergence guaranteed in some important cases
  5. Final probability distribution solves efficient influence curve equation
     - basis for asymptotic linearity, normality, and efficiency.
     - confers double robustness, or, more general, makes bias a second order term.
- Asymptotically efficient when initial and treatment/censoring mechanism estimators are both consistent
- Allows incorporation of machine learning while preserving inference

TMLE Algorithm for our ATE example

- Step 1: Obtain initial estimate

$$\bar{Q}_n^0(A, W) = \hat{E}(Y \mid A, W)$$

- Step 2: Target initial estimate (logit scale)

$$\bar{Q}_n^*(A, W) = \bar{Q}_n^0(A, W) + \hat{\epsilon} h_{g_n}(A, W)$$

  - Estimate $g_0(A, W)$ (propensity score)
  - Construct parameter-specific fluctuation covariate, e..g,

$$h_{g_n}^{ATE} = \left[\frac{A}{g_n(1, W)} - \frac{1 - A}{g_n(0, W)} \right]$$

  - Maximum likelihood to fit ϵ

- Evaluate parameter: $\psi_n^{TMLE} = \Psi(\bar{Q}_n^*)$

Efficient Influence Curve for ATE

TMLE solves $P_n D^*(P_n^*) = 0$

- Efficient influence curve for ATE parameter

$$D^*(P) = \underbrace{\left[\frac{A}{g(1,W)} - \frac{1-A}{g(0,W)}\right]\left[Y - \bar{Q}(A,W)\right]}_{a} + \underbrace{\bar{Q}(1,W) - \bar{Q}(0,W) - \psi}_{b}$$

- Stage 2 targeting fits ϵ by maximum likelihood
  - MLE solves score equation $\sum_i h_{g_n}(A_i, W_i)[Y_i - \bar{Q}_n^*(A_i, W_i)] = 0$
  - We define parameter-specific h_g to ensure that the empirical mean of a equals 0.
- As a substitution estimator $\psi_n^{TMLE} = \frac{1}{n}\sum_i \bar{Q}_n^*(1, W_i) - \bar{Q}_n^*(0, W_i)$, thus empirical mean of b equals 0.

Inference

- Asymptotic Linearity

$$\sqrt{n}(\psi_n^{TMLE} - \psi_0) \xrightarrow{D} N(0, \sigma^2)$$

- 95% confidence intervals

$$\psi_n(Q_n^*) \pm 1.96\, \hat{\sigma}/\sqrt{n}$$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} \hat{D}^{*^2}(P_n^*)(O_i)$$

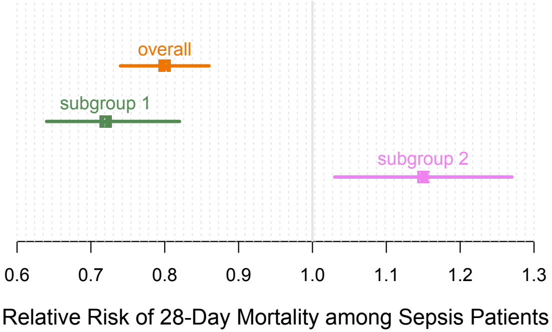
- Test statistic for null hypothesis H_0: $\psi_0 = 0$

$$T = \frac{\psi_n}{\sqrt{\hat{\sigma}^2/n}}$$

- p-values are calculated as $2\Phi(-abs(T))$
  Φ = CDF of standard normal distribution

Effect of Steroids on Mortality in Adults with Septic Shock

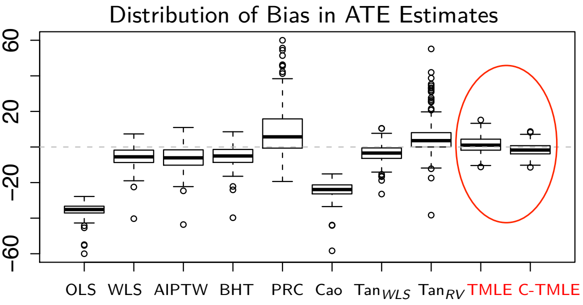
- Patient-level data from three major randomized controlled trials*
- Conflicting results among the three studies
- Using TMLE + SL to estimate RR of mortality (steroid vs. placebo)
  - Reduced variance
  - Power to estimate significant heterogeneous subgroup effects



*Pirracchio, et. al. under review

Simulation Study: ATE parameter estimation

- TMLE and C-TMLE compared with ordinary least squares and seven other double robust estimators
- Challenges: Misspecified outcome regression, correct propensity score model produces near positivity violations
- TMLE and C-TMLE were least biased and had smallest variance



Porter, Gruber, Sekhon, and van der Laan. *The International Journal of Biostatistics*, 2011.

Additional topics

- Point treatment parameters
  - Relative risk, odds ratio, risk difference
  - Mean outcome under missingness in the population
  - Effect of treatment among the treated (ATT)
  - Controlled direct effects
  - Marginal structural model parameters
- Case-control and other biased sampling designs
- Addressing common challenges in data analysis
  - Near positivity violations (poor overlap): Collaborative TMLE (C-TMLE) for data-adaptive nuisance parameter estimation
  - TMLE for bounded continuous outcomes
  - TMLE for rare outcomes

TMLE Demonstration

- *tmle* R package (CRAN)
- Using the package
  - Examples taken from: S. Gruber and MJ van der Laan. *tmle*: An R Package for Targeted Maximum Likelihood Estimation. Journal of Statistical Software 2012; 51(13)
- Practical Considerations
  - Setting bounds on Q
  - Truncation level for g
  - Examining results
    - Summary
    - Obtaining untargeted parameter estimates